## Project Summary: Finite Element Analysis for Options Pricing (Focus on Call Options)

Option Contract: A contractual right but not an obligation to purchase (call option) or sell (put option) an asset (typically in bundles of 100 stocks) at a certain strike price by a certain maturity date. European option contracts only allow the contract to be exercised at the maturity date. American option contracts allow the contract to be exercised prior to the maturity date as long as the option is in-the-money or at-the-money.

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| --- | --- | --- |
|  | Call Option | Put Option |
| In-the-money (ITM) | Stock Price (S) > Strike Price (K) | S < K |
| Out-of-the-money (OTM) | S < K | S > K |
| At-the-money (ATM) | S = K | S = K |

Option contracts expire worthless if it is OTM at maturity and are automatically exercised if ITM/ATM. Buying and selling option contracts are allowed at any time the market is open.

Black-Scholes PDE: A PDE that is used to determine European options pricing. Works for both calls and puts.

|  |  |
| --- | --- |
| Classical Black-Scholes PDE: |  |
| Transformed Black-Scholes PDE: |  |

The classical Black-Scholes PDE has top, bottom, and terminal boundary conditions. To convert the terminal condition to an initial condition, define , and the PDE becomes:

Boundary conditions for call option contracts:

, ,

Boundary conditions for put option contracts:

, ,

The boundary conditions use the time value of money and assumes continuous compounding. The difference between the FE model of call option and a put option are the boundary conditions. Focus of this project is on call options.

Three models were created and compared, also with a theoretical solution:

1. Finite difference model using the classical Black-Scholes PDE and applying central differencing to the stock price dimension and Crank-Nicolson to the time dimension
2. Finite element model using the classical Black-Scholes PDE and applying 1D Galerkin Method of Weighted Residuals (GMWR) to the stock price dimension and Crank-Nicolson to the time dimension
3. Finite element model using the transformed Black-Scholes PDE and applying 1D GMWR to the stock price dimension and Crank-Nicolson to the time dimension

Made a mistake on the second model and cannot find the mistake. It is wrong.

Theoretical solution:

,

Example mesh using model 1:

Chart, surface chart

Description automatically generated



Example mesh using model 3:

Chart, surface chart

Description automatically generated



For model 3, the boundary conditions are modified to be in terms of F and x, and they must be converted back to V and S when plotting the mesh. All meshes plotted with , not t.

Transformation Variables: , ,

BCs: , ,

For some reason, the finite difference model is more accurate than the finite element model. Online sources seemed to have found the same trend (sanity check).